Teleportation with trapped ions in a magnetic field gradient

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Abstract

By means of the Ising terms generated by Coulomb interaction between ions and the magnetic field gradient, we carry out teleportation with insurance with trapped ions. We show the feasibility and the favorable feature of our scheme by comparing with the recently achieved teleportation experiments with trapped ions.

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I. INTRODUCTION

The trapped-ion system is one of the most promising candidates for quantum information processing (QIP), which has achieved significant experimental advances, for example, entanglement of few ions [1], realization of simple quantum gates or quantum algorithms [2, 3, 4] and so on. Particularly, the recently achieved teleportation [5, 6] in ion traps paves the way towards scalable QIP with trapped ions.

Much effort has been paying on overcoming barriers of scalability and fidelity, such as decoherence due to heating, imprecise laser focusing and instability of operation at optical frequencies [7]. We have noticed some technical improvement in experiments as well as some potentially practical proposals. In this Letter, we focus on a long wave-length scheme, which corresponds to a modified ion trap with a magnetic field gradient applied to the confined ions [8, 9, 10]. In this scheme, the qubits are encoded in the atomic hyperfine levels, and due to the magnetic field gradient the ions can be distinguished in frequency space and individually addressed by microwaves. This new scheme is advantageous over those in original ion traps in following points: (1) Raman-type radiation is not necessary. So the qubit rotation carried out by microwaves is straightforward. (2) Ising terms due to the magnetic field gradient give the possibility of quantum gating by the well developed NMR-type implementation. (3) Quantum computing can be made with an array of ions confined in either a linear trap or some individual micro-traps, where the spin-spin coupling strength between different ions is adjustable in the latter case. (4) The cheap source of microwave makes experiments performed more easily than lasers.

In this Letter, we study how to carry out teleportation based on the models in Refs. [8, 9, 10]. Our calculation starts from the multi-trap model in [10]. Since three ions confined in a linear trap can be considered mathematically as a special situation of the multi-trap model, i.e., the same trap frequency for each of the ions, our discussion will cover the three-ion cases in either a single trap or three traps respectively. As shown in Fig.1, we first consider three $^{171}\mathrm{Yb}^{+}$ ions, each confined in an individual micro-trap. The hyperfine levels ($|1\rangle$ =|F=1, m_F=+1> and $|0\rangle$ =|F=0>) with qubits encoded are split differently for different ions. To avoid heating, we will carry out teleportation only by carrier transitions by using the method in NMR quantum computation. Comparing with the achieved teleportation experiments, we show the merits of our scheme and discuss whether it could be achieved by

current techniques.

II. THE CONFIGURATION OF THREE IONS

For a system with three ions in Fig. 1, the coupling strength between different ions can be adjusted by choosing the position and the trap frequency of each trap appropriately. According to Ref. [10], we have in units of $\hbar = 1$,

$$H_0 = \sum_{i=1}^{3} \frac{1}{2} w_i(z_{0,i})\sigma_{z,i} - \frac{1}{2} J\sigma_{z,1}\sigma_{z,2} - \frac{1}{2} J\sigma_{z,2}\sigma_{z,3} - \frac{1}{2} J_{13}\sigma_{z,1}\sigma_{z,3} + \sum_{\ell=1}^{3} \nu_\ell a_\ell^+ a_\ell, \tag{1}$$

where $J_{ij} = \sum_{\ell=1}^{3} \frac{\hbar}{2m\nu_\ell^2} D_{i,\ell} D_{j,\ell} \frac{\partial w_i}{\partial z}|_{z_{0,i}} \frac{\partial w_j}{\partial z}|_{z_{0,j}}$ with D a unitary matrix related to the Hessian of the potential, $w_i(z_{0,i})$ is the position-dependent transition frequency for ion i in the magnetic field gradient with $z_{0,i}$ the equilibrium position, ν_ℓ is the ℓth vibrational frequency, and σ_{zi} is the usual Pauli operator for ion i. We have adjusted $J_{12} = J_{23} = J$ by simply setting the trap frequencies of micro-traps for ions 1 and 3 to be equal. To avoid exciting the vibrational mode, we will only consider the carrier transition. So we neglect the last term in Eq. (1). Direct algebra shows that in the space spanned by $|0\rangle_1|0\rangle_2|0\rangle_3$, $|1\rangle_1|0\rangle_2|0\rangle_3$, $|0\rangle_1|1\rangle_2|0\rangle_3$, $|0\rangle_1|0\rangle_2|1\rangle_3$, $|1\rangle_1|1\rangle_2|0\rangle_3$, $|1\rangle_1|0\rangle_2|1\rangle_3$, $|0\rangle_1|1\rangle_2|1\rangle_3$, $|1\rangle_1|1\rangle_2|1\rangle_3$, the eigenenergies are $-\frac{w_1}{2} - \frac{w_2}{2} - \frac{w_3}{2} - J - \frac{J_{13}}{2}$, $\frac{w_1}{2} - \frac{w_2}{2} - \frac{w_3}{2} + \frac{J_{13}}{2}$, $-\frac{w_1}{2} + \frac{w_2}{2} - \frac{w_3}{2} + J - \frac{J_{13}}{2}$, $-\frac{w_1}{2} - \frac{w_2}{2} + \frac{w_3}{2} + \frac{J_{13}}{2}$, $\frac{w_1}{2} + \frac{w_2}{2} - \frac{w_3}{2} + \frac{J_{13}}{2}$, $\frac{w_1}{2} - \frac{w_2}{2} + \frac{w_3}{2} + J - \frac{J_{13}}{2}$, $-\frac{w_1}{2} + \frac{w_2}{2} + \frac{w_3}{2} + \frac{J_{13}}{2}$, $\frac{w_1}{2} + \frac{w_2}{2} + \frac{w_3}{2} - J - \frac{J_{13}}{2}$, respectively. As shown in Fig. 2, the carrier transition frequencies for each ions are dependent on the states of the other two ions [11].

In Table I, under the restriction $\varepsilon_{\max} < 0.05$($\varepsilon_{\max}$ is the maximum of all $\varepsilon_{i,\ell}$ defined in the next section), we list different cases in our consideration, in which the trap frequencies and magentic field gradient are chosen for getting the biggest J with respect to a certain neighboring trap distance d. It's obvious that the smaller the neighboring trap distance d, the bigger the J obtainable. Moreover, to resonantly excite a certain ion irrespective of the states of the other two ions, we should have a microwave with bandwidth larger than the maximum difference between the carrier transition frequencies regarding this ion. In what follows, we will use the numbers in Table I regarding d=4μm to demonstrate our scheme. If the microwave bandwidth is $63.7 \times 2\pi kHz$, as we can see in Fig.2,

we cannot distinguish different carrier transitions with this microwave for individual ions. Straightforward calculation also shows that the three vibrational frequencies are approximately $1.32 \times 2\pi MHz, 1.54 \times 2\pi MHz, 1.70 \times 2\pi MHz$, and the resonance frequency shift between neighboring qubits is $g\mu_B \frac{\partial B}{\hbar \partial z}(d + \Delta) \approx 64.8 \times 2\pi MHz$ (Δ represents the distance between equilibrium position of ion 1 or ion 3 and their own trap centers.)

III. CONTROLLED-NOT GATE

The controlled-NOT (CNOT) gate is the most commonly used two-qubit quantum gate. When a microwave is added, the interaction Hamiltonian in our system differs from that in original ion trap only by replacing the Lamb-Dicke parameter $\eta_{i,\ell}$ with $\eta'_{i,\ell} = \sqrt{\eta_{i,\ell}^2 + \varepsilon_{i,\ell}^2}$, where the subscript i denotes ion i, ℓ is for the ℓth collective motional mode, $\eta_{i,\ell}$ is about 10^{-6}, and $\varepsilon_{i,\ell} = D_{i,\ell}(\sqrt{\hbar/2m\nu_\ell}\frac{\partial w_i}{\partial z}|_{z_{0,i}})/\nu_\ell$ is an additional Lamb-Dicke parameter due to the magnetic field gradient [9]. In the Paschen-Bach limit ($B_0 \sim 1$), as the frequency gradients are independent of z, $\frac{\partial w_i}{\partial z}|_{z_{0,i}} = \frac{2}{\hbar}\mu_B \frac{\partial B}{\partial z}$[10].Given the numbers in Table I for d=4μm, the maximum $\varepsilon_{i,\ell}$ is about 0.0340. So all the $\eta'_{i,\ell}$ are much smaller than 1, and we can get to a reduced unitary evolution operator for addressing ion i by a microwave in the interation picture,

$$U_I^i(\theta, \phi) = \exp[i\frac{\theta}{2}(e^{-i\phi}\sigma_+ + e^{i\phi}\sigma_-)], \tag{2}$$

where $\theta = \Omega t$ with Ω the Rabi frequency, and ϕ is related to the position of the ion i in the microwave. We suppose Ω to be of the order of MHz. If $\phi = 0$, then $U_I = \exp(i\frac{\theta}{2}\sigma_x)$, and if $\phi = \frac{\pi}{2}$, then we have $U_I = \exp(i\frac{\theta}{2}\sigma_y)$.

In NMR quantum computation, CNOT gate is realized by a sequence of pulses as below [12],

$$U_{cnot}(i,j) = e^{-i\pi/4} e^{-i\pi/4\sigma_{y,j}} e^{+i\pi/4\sigma_{z,i}} e^{+i\pi/4\sigma_{z,j}} e^{-i\pi/4\sigma_{z,i}\sigma_{z,j}} e^{+i\pi/4\sigma_{y,j}}, \tag{3}$$

where ions i and j act as control and target qubits, respectivley. $e^{-i\pi/4\sigma_{z,i}\sigma_{z,j}}$ can be realized by means of the coupling term $-\frac{1}{2}J\sigma_{z,i}\sigma_{z,j}$ in H_0 and refocusing techniques must be used to eliminate undesired evolution brought by the other terms in H_0 [13]. All the other terms in Eq. (3) can be implemented by microwave pulses. But we note that, in the context of NMR, Rabi frequency is much larger than other characteristic frequencies. In our case,

however, $w_i(z_{0,i}) \sim 13 \times 2\pi$GHz is the biggest frequency, much bigger than Rabi frequency. In order to avoid undesired evolution due to $w_i(z_{0,i})$, we need carefully control the pulse length T to satisfy

$$w_i(z_{0,i})T = 2\pi N(i = 1, 2, 3; N = 1, 2, 3....), \tag{4}$$

which can be done by properly adjusting B_0, $\partial B/\partial z$ and Ω. In Table II, we give an example of $U_{cnot}(2, 3)$. The total time for completing a CNOT gate is about $3.84ms$. If we decrease the neighboring trap distance d, the CNOT gating time can be shortened due to the enlarged J.

IV. TELEPORTATION

Teleportation [14] is a disembodied transport of unknown quantum states by combining quantum and classical channels. The two separated partners Alice and Bob share two entangled particles 2 and 3 initially. After Alice inputs her information by making joint measurement on particles 1 and 2, Bob is able to recover the state of particle 1 in particle 3, assistent with some unitary transformations conditional on the classical communication with Alice. Since its proposal, teleportation has been experimentally realized in both macroscopic [15] and microscopic distances [5, 6, 16]. In what follows, we show how to carry out a teleportation in our system.

Considering ion 1 initially in an arbitrary state $\alpha|0\rangle_1 + \beta|1\rangle_1$, ions 2 and 3 in states $\frac{1}{\sqrt{2}}(|0\rangle_2 + |1\rangle_2)$ and $|1\rangle_3$ respectively, after the CNOT gate $U_{cnot}(2, 3)$, we get the entangled state $\frac{1}{\sqrt{2}}(|0\rangle_2|1\rangle_3 + |1\rangle_2|0\rangle_3)$ whose lifetime can be longer than 100ms [5]. So at this stage the three ions are in a state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle_1 + \beta|1\rangle_1)(|0\rangle_2|1\rangle_3 + |1\rangle_2|0\rangle_3). \tag{5}$$

With another c-not gate $U_{cnot}(1, 2)$, we have

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|0\rangle_2|1\rangle_3 + |1\rangle_2|0\rangle_3) + \beta|1\rangle_1(|1\rangle_2|1\rangle_3 + |0\rangle_2|0\rangle_3)]. \tag{6}$$

Then a Hadamard transformation $|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ on ion 1 would yield

$$\begin{aligned}|\Psi_3\rangle = \frac{1}{2}[&|0\rangle_1|0\rangle_2(\alpha|1\rangle_3 + \beta|0\rangle_3) + |0\rangle_1|1\rangle_2(\alpha|0\rangle_3 + \beta|1\rangle_3) \\ &+ |1\rangle_1|0\rangle_2(\alpha|1\rangle_3 - \beta|0\rangle_3) + |1\rangle_1|1\rangle_2(\alpha|0\rangle_3 - \beta|1\rangle_3)].\end{aligned} \tag{7}$$

Therefore, similar to Refs. [5, 6], by making measurement on ions 1 and 2 in the bases $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$, we can obtain $\alpha|0\rangle_3 + \beta|1\rangle_3$ assistent with appropriate unitary qubit rotation σ_x, I, $i\sigma_y$, σ_z. All these operations can be achieved by microwave pulses by using Eq. (2). Because all the four bases can be distinguished definitely, our teleportation is done with insurance as in Refs. [5, 6].

V. DISCUSSION AND CONCLUSION

Since the three ions under our consideration are confined in separate micro-traps, our calculation above is actually for a multi-trap system. However, different from the multiplexed traps in Ref. [6], the magnetic field gradient in our case provides spin-spin couplings between the ions. So we can entangle the ions without using vibrational mode as bus qubit or detecting the leaking photons [17].

As the ions are distinguished in frequency space under the magnetic field gradient, we can carry out teleportation without moving or hiding any ions, which is much simpler than in Refs. [5, 6]. Due to the spin-spin coupling in our system, we accomplish quantum gates by the way similar to NMR quantum computing. This is different from the Cirac-Zoller CNOT gate in Ref. [5], which has to excite the vibrational mode, and is also different from the geometric phase gating in Ref. [6]. Although in the latter work the geometric phase gate was made by only virtually exciting the vibrational mode, like we are doing here, the movement of the ions between different traps, which inevitably yields heating, reduces the fidelity of the teleported state. Besides, the required bichromatic radiation in Ref. [6], based on the Raman process, makes the experiment very challenging. In contrast, our method, without ions' moving and by using the sophisticated microwave pulse sequenses, is more straightforward.

However, the teleportation time in our case is longer than those in Refs. [5, 6] by approximately two times, which is not good in view of decoherence. To reduce the implementation time, we have to increase the Ising interacion J, which can be done by reducing the inter-ion spacing d or enlarging the magnetic field gradient. However, the smaller the inter-ion distance, the more challenging the experimental implementation, i.e., more focused laser beams or higher quality micro-traps are needed. Besides, the bigger magnetic field gradient would increase the effective Lamb-Dicke parameter. Since the Zeeman splitting of the ions are always changing due to the vibration of the ions in the magnetic field gradient, to reduce infidelity, we require the ions keeping strictly within the Lamb-Dicke limit throughout the our scheme.

Although we neglect the vibrational modes, since the Ising term is from the virtually coupling to the vibrational modes, we still need to consider heating of the vibrational mode in our two-qubit gating. We know the heating time of the ground vibrational mode to be of the order of 4 ms for a trap of the size of $\mathbf{100}\mu$m [6]. Since the heating rate scales with the trap size R as R^{-4} [18], the micro-trap in our considerartion, which is of the size of 4 μm, would own a heating time of the order of nanosec. Therefore our multi-trap scheme is not achievable with current technique. Nevertheless, if we turn to a linear trap with three ions confined, which corresponds to the treatment in Refs. [8, 9] and also to the situation in Ref. [5] but with a magnetic field gradient applied, we find the feasibility of our scheme. Based on Table III, we obtain the CNOT gating time is about 4.94 ms for the three ions with the neighboring spacing of 4 μm. Suppose in our case the heating time to be 1 ms, and only 10% vibrational mode being actually excited in our two-qubit gate, the actual heating time would be 10 ms, long enough for our scheme.

Experimentally, three ions in a linear trap with the spacing of 4-5 μm has been achieved [5], and the microwave with a certain bandwidth and a rapid change of phase ϕ is a sophisticated technique. Moreover, magnetic field gradient up to 8000 T/m is experimentally available in the near future [8]. We should mention here that in this long wave-length model lasers are still necessary to provide the initially cooling of ions, initial state preparation and final read-out, as mentioned in [8, 9, 10]. Therefore, the ions are required to be distingushed in the optical frequency domain, which can be achieved by beam-focusing technique [3, 5].

Compared with the teleportation by photons [19], our implementation is deterministic, and the trapped ions are better candidates for storing information than the flying photons.

Moreover, although we make use of the NMR-type inplementation, our gating is really performed on individual quantum states, instead of spin ensemble in NMR system [16]. So what we show here is really a teleportation of a quantum state. Furthermore, we noticed the Ising coupling widely used in solid state QIP, such as in semiconductor quantum dots [20], in doped fullerenes [11, 21], and so on. Quantum gating in these solid-state systems is still very challenging experimentally [22]. Therefore, the relatively easier achievement of the CNOT and teleportation in our trapped ions system would provide a possibility to test different QIP proposals for above solid-state systems.

In summary, we have specifically investigated a teleportation scheme with three trapped ions in a magnetic field gradient. We show the possibility to realize teleportation without moving any ions and without exciting vibrational modes. By considering currently available ion traps, we argue that our scheme is achievable in a linear trap and will also be feasible in multi-trap devices in the future if the heating problem can be overcame. Although the different energy splittings of the ions in our system due to magnetic field gradient require more attention to the refocusing, we don't think that those operations would be more technically difficult than in original ion traps, as long as we know exactly the energy splittings of each qubits and the ions are strictly restricted within the Lamb-Dicke limit.

ACKNOWLEDGMENTS

The authors acknowledge thankfully the helpful discussion with Derek Mc Hugh, Jun Luo and Christof Wunderlich. MF is also grateful to Jason Twamley for his hospitality and encouragement when he stayed in NUIM. Z.J. D is thankful for warmhearted help from Guilong Huang, Baolong Lu, Daxiu Wei and Xiwen Zhu. This work is supported by National Natural Science Foundation of China under contract numbers 10474118 and 10274093.

Note added After finishing this Letter, we become aware of a work [23], in which the Ising coupling between ions is obtained by changing the laser intensities and the polarizations. As it is mathematically identical to the models in [8, 9, 10], our scheme can be in principle applied to it.

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Figure Captions

Fig. 1. Schematic diagram for three $^{171}Yb^+$ ions, each confined in an individual micro-trap. The magnetic field gradient is applied along the z axis (i.e. the ion array). So the energy differences of the ions are position-dependent. d is the separation between neighboring traps and $\triangle$ denotes the deviation of the equilibrium position for ion 1 or ion 3 from their respective trap center.

Fig. 2. Spectrum of the carrier transition frequencies, where each vertical line represents a carrier transition frequency of a certain ion with corresponding states of the other two ions labeled below.

Table I. Cases of three ^{171}Yb$^+$ having the largest J with respect to a certain neighboring trap distance d, where W_1 is the trap frequency for ion 1 and ion 3, W_2 is the trap frequency for ion 2, $\partial B/\partial z$ denotes magnetic field gradient and Δ represents the distance between equilibrium position of ion 1 or ion 3 and their own trap center and h is the inter-ion distance.

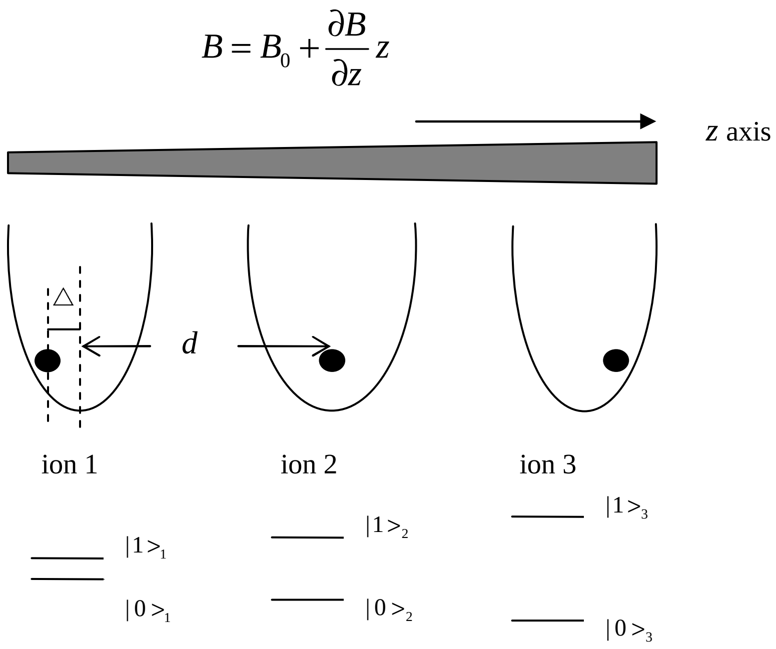
$d(\mu m)$	$W_1(MHz)$	$W_2(MHz)$	$\partial B/\partial z(T/m)$	$\Delta(\mu m)$	$\varepsilon_{\max}$	$h(\mu m)$	$J(kHz)$	$J_{13}(kHz)$
1	$3.20\times 2\pi$	$0.097\times 2\pi$	1200	0.779	0.0376	1.779	$1.60\times 2\pi$	$0.746\times 2\pi$
2	$2.72\times 2\pi$	$1.27\times 2\pi$	1000	0.531	0.0422	2.531	$1.07\times 2\pi$	$0.337\times 2\pi$
3	$1.85\times 2\pi$	$1.10\times 2\pi$	600	0.578	0.0427	3.578	$0.645\times 2\pi$	$0.179\times 2\pi$
4	$1.37\times 2\pi$	$1.24\times 2\pi$	500	0.628	0.0340	4.628	$0.459\times 2\pi$	$0.135\times 2\pi$
5	$1.21\times 2\pi$	$1.05\times 2\pi$	400	0.558	0.0382	5.558	$0.334\times 2\pi$	$0.0820\times 2\pi$
6	$0.971\times 2\pi$	$0.891\times 2\pi$	300	0.612	0.0356	6.612	$0.254\times 2\pi$	$0.0623\times 2\pi$
7	$0.732\times 2\pi$	$0.700\times 2\pi$	200	0.777	0.0319	7.777	$0.197\times 2\pi$	$0.0515\times 2\pi$

Table II. The implementation pulses and time for each term of $U_{cnot}(2,3)$, where U_0 denotes the corresponding unitary evolution operator of H_0 (without including the last term in Eq. (1)), X_i^2 means $U_I^i(\pi,0)$, and we adopt the numbers in Table I for d=4μm. For simplicity, we assume the implementation time for any single qubit rotation $U_I^i(\theta,\phi)$ to be $t_m = 2.5\mu s$.

term	implementation pulses	time	estimated time	total estemated time
$e^{+i\pi/4\sigma_{y,3}}$	$U_I^3(\frac{\pi}{2},\frac{\pi}{2})$	t_m	$2.5\mu s$	$3.84ms$
$e^{-i\pi/4\sigma_{z,2}\sigma_{z,3}}$	$X_3^2X_2^2U_0(\frac{t}{4})X_1^2U_0(\frac{t}{4})X_3^2X_2^2$ $U_0(\frac{t}{4})X_1^2U_0(\frac{t}{4})(\mathbf{t}=\frac{\mathbf{7\pi}}{\mathbf{2J}})$	$t+4t_m$	$3.82ms$	
$e^{+i\pi/4\sigma_{z,3}}$	$U_I^3(\frac{\pi}{2},\frac{\pi}{2})U_I^3(\frac{\pi}{2},0)U_I^3(\frac{7\pi}{2},\frac{\pi}{2})$	$3t_m$	$7.5\mu s$	
$e^{+i\pi/4\sigma_{z,2}}$	$U_I^2(\frac{\pi}{2},\frac{\pi}{2})U_I^2(\frac{\pi}{2},0)U_I^2(\frac{7\pi}{2},\frac{\pi}{2})$	$3t_m$	$7.5\mu s$	
$e^{-i\pi/4\sigma_{y,3}}$	$U_I^3(\frac{7\pi}{2},\frac{\pi}{2})$	t_m	$2.5\mu s$	

Table III. Cases of three ^{171}Yb$^+$ in a linear trap with the largest J (in our calculation) with respect to a certain inter-ion distance h, where W is the trap frequency, $\partial B/\partial z$ denotes magnetic field gradient.

$h(\mu m)$	$W(MHz)$	$\partial B/\partial z(T/m)$	$\varepsilon_{\max}$	$J(kHz)$	$J_{13}(kHz)$
2	$1.77 \times 2\pi$	750	0.0276	$1.12 \times 2\pi$	$0.794 \times 2\pi$
3	$0.966 \times 2\pi$	300	0.0271	$0.605 \times 2\pi$	$0.429 \times 2\pi$
4	$0.628 \times 2\pi$	150	0.0263	$0.359 \times 2\pi$	$0.254 \times 2\pi$
5	$0.449 \times 2\pi$	100	0.0289	$0.311 \times 2\pi$	$0.220 \times 2\pi$
6	$0.342 \times 2\pi$	50	0.0218	$0.134 \times 2\pi$	$0.0952 \times 2\pi$



$B = B_0 + \frac{\partial B}{\partial z} z$
z axis
$\triangle$
d
ion 1
ion 2
ion 3
$|1>_1$
$|0>_1$
$|1>_2$
$|0>_2$
$|1>_3$
$|0>_3$

Fig. 1.

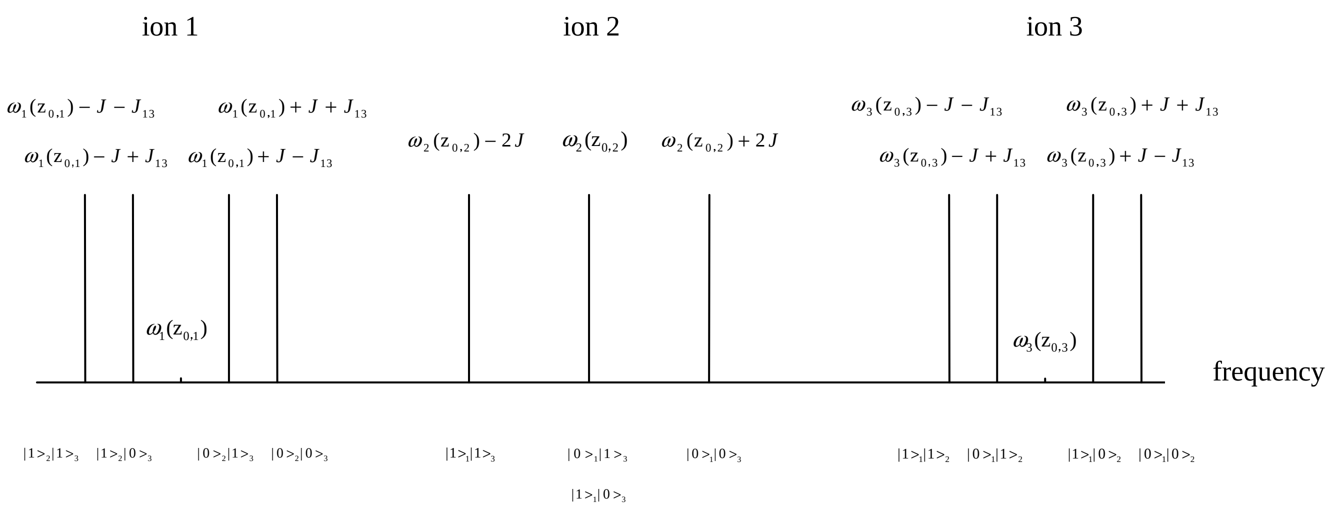

ion 1
ion 2
ion 3
$\omega_1(z_{0,1}) - J - J_{13}$
$\omega_1(z_{0,1}) + J + J_{13}$
$\omega_1(z_{0,1}) - J + J_{13}$
$\omega_1(z_{0,1}) + J - J_{13}$
$\omega_2(z_{0,2}) - 2J$
$\omega_2(z_{0,2})$
$\omega_2(z_{0,2}) + 2J$
$\omega_3(z_{0,3}) - J - J_{13}$
$\omega_3(z_{0,3}) + J + J_{13}$
$\omega_3(z_{0,3}) - J + J_{13}$
$\omega_3(z_{0,3}) + J - J_{13}$
$\omega_1(z_{0,1})$
$\omega_3(z_{0,3})$
frequency
$|1>_2|1>_3$
$|1>_2|0>_3$
$|0>_2|1>_3$
$|0>_2|0>_3$
$|1>_1|1>_3$
$|0>_1|1>_3$
$|1>_1|0>_3$
$|0>_1|0>_3$
$|1>_1|1>_2$
$|0>_1|1>_2$
$|1>_1|0>_2$
$|0>_1|0>_2$

Fig. 2.